Resonance of Air Columns

1. Theory

Imagine two waves, say simple harmonic motion (SHM, *i.e.*, sine/cosine) waves of identical wavelength and amplitude traveling in opposite directions with equal speeds. The net displacement of the medium at any point and at any time is determined by applying the superposition principle which states that the net displacement is given by the algebraic sum of the two individual displacements. The resulting wave pattern will then have points, separated by one-half wavelength, where the displacement is always zero. These points are called nodes. Midway between this nodes, the particles of the medium located at the antinodes, vibrate with maximum displacement.

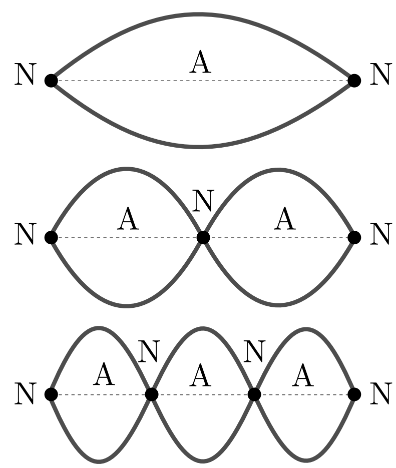

Fig. 1: The three lowest frequency for transverse vibrations of a string clamped at the ends.

We can visualize transverse standing waves on a string, of length L, fixed at both ends. These waves can be established by plucking the string at some point and are caused by continual reflection of the traveling waves at the boundaries, in this case the two fixed ends. The boundary conditions demand that at each end there must be a node. We can therefore fit an integral number of half-wavelengths into the length L of the string as shown in Fig. 1.

Even though the wave shape is not moving, we can associate a velocity with the standing wave which is the same as that of the traveling wave in the same medium. It can be deduced from Fig. 1, that the wavelengths of the traveling waves that combine to give the standing waves are given by $L = \frac{n(\lambda_n)}{2}$ so that $\lambda_n = \frac{2L}{n}$, where n is an integer. The corresponding frequencies are therefore given by $f_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = nf_0$ with $f_0 = \frac{v}{2L}$, where v is the speed of transverse traveling waves on the string.

The frequencies are known as resonance frequencies of this system. The lowest frequency of the system, having the longest wavelength, is called the fundamental frequency and the mode of vibration as the fundamental mode or the first harmonic. The modes of vibration with progressively higher frequencies, are called second harmonic ($n = 2$), third harmonic ($n = 3$), *etc.*

Longitudinal standing waves of air columns can also be set up in closed (open at one end, closed at the other) and in open (open at both ends) organ pipes. A longitudinal wave, such as a sound wave, is a wave of density variation. The fixed or closed and of the pipe cannot have any longitudinal motion and is therefore a node of the density wave. The free or open end of the pipe is a position of maximum longitudinal displacement and is therefore an antinode of the density wave.

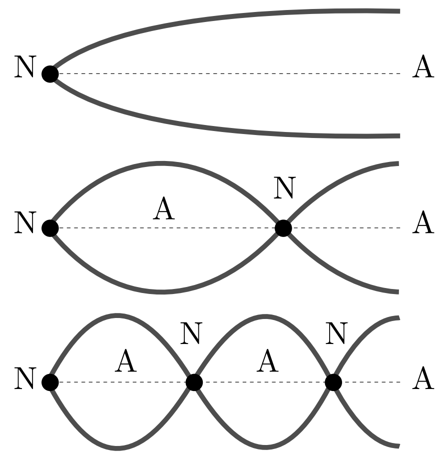

Fig. 2: The longitudinal vibrations of air in a pipe, closed at the left and open at the right.

In a closed pipe of length L the boundary conditions require that the closed end be a node and the open end an antinode. We can therefore fit an odd multiple of quarter-wavelengths into the length L of the pipe as shown in Fig. 2.

The transverse wave pattern for the standing wave for the standing waves are displayed in this figure. The wavelengths of the traveling waves that combine to give the standing waves are given by $L = \frac{n\lambda}{4}$, *i.e.*, $\lambda = \frac{4L}{n}$ where $n = 1, 3, 5, ...$ The corresponding resonance frequencies are therefore given by $f_n = \frac{v}{\lambda_n} = \frac{nv}{4L} = nf_0$ with $f_0 = \frac{v}{4L}$, where v is the speed of longitudinal traveling waves(sound) in air. It is to be noted that the closed organ pipe will support only odd harmonics. The frequency of the first harmonic or the fundamental frequency of equals $(\frac{v}{4L})$.

For an organ pipe in which both ends of the pipe are open the situation is the same as for a string with its two ends fixed except that both the ends are now antinodes for any standing harmonic wave. The wave patterns therefore will yield the resonance frequencies as $f_n = n\frac{v}{2L} = nf_0$ with $f_0 = \frac{v}{2L}$. It is seen that the open organ pipe will support all harmonics.

For a given frequency, the first resonance corresponds to the length of the closed-end air column equal to $\frac{\lambda}{4}$. The second resonance corresponds to $\frac{3\lambda}{4}$. The difference between these positions would correspond to half a wave length. The sound velocity is determined from $v = f\lambda$ where f is the tuning fork frequency in Hertz. At room temperature T (in degrees of Celsius), the theoretical value of sound velocity (in m/s) is $v = 331+0.61T$.

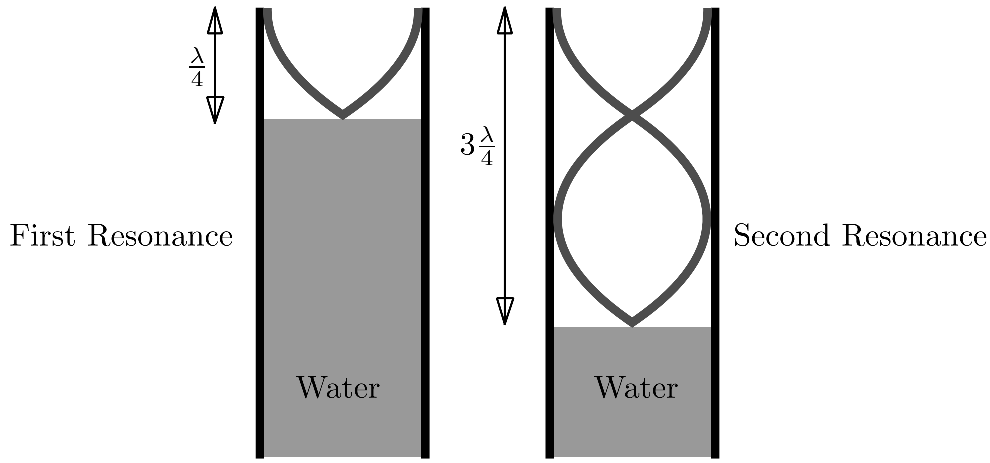

Fig. 3: The longitudinal vibrations of an air column, in a pipe with water at the bottom.

2. Experiment

Object: To determine the speed of sound in air using resonance of a close-end air column.

Apparatus: Resonance tube apparatus, tuning fork, rubber mallet and a Macintosh Computer.

Procedure:

1. Strike a tuning fork of frequency 512 Hz with a rubber mallet and hold it at about an inch above the open end of the resonance tube with its prongs horizontal. Adjust the water level starting from its highest level. Gradually increase the length of the air column by lowering the can to find the first position of resonance, where the sound coming out of the air column is loudest. You may have to strike the fork several times and move the water column up and down to precisely locate the resonance position.
3. Continue this procedure to find second (and if possible, the third) position of resonance. Record these lengths as l_1 and l_2.
4. Repeat the experiment with a tuning fork of different frequency.

Calculations: Using the computer spreadsheet perform the following calculation.

1. Calculate the speed of sound from the formula $v = \lambda f = 2(l_2 - l_1)f$, where f is the frequency of the tuning fork.
2. If you get only the first resonance, and not the second resonance then, to calculate the speed of the sound, use $v = 4fl_1$.
3. Compare the calculated speed of sound with the theoretical value from the formula $v = 331 + 0.61T$, where T is the room temperature in Celsius degrees.

Questions:

1. Sketch the air column vibrations for third resonance. What is the difference between the positions of the third and the first resonance in terms of wave length?
2. Should the velocity of sound in air depend upon the frequency of the tuning fork?

3. Are there "anti-resonances" where the sound coming out of the air column reaches minimum? What is the length of air columns for these "anti-resonances"?
4. Why does the resonance position correspond to the "loudest sound"?
5. Do you see any advantage of $v = f(l_2 - l_1)$ over $v = 4fl_1$?