Simple Harmonic Motion

1-Theory

Periodic motion or harmonic motion is motion that repeats itself at regular intervals of time. The simplest of all periodic motion is called simple harmonic motion (SHM). This motion is represented by a sinusoidal function of time. Mathematically it may be written as

$$y(t) = y_m \cos(\omega t + \Phi), \qquad\qquad (1)$$

where $y(t)$ is the displacement of the particle from its equilibrium position at time t, y_m is the amplitude (maximal displacement), ω is the angular frequency, and the constant Φ is called the phase angle.

Imagine a spring (unstretched length l), that is hanging vertically from a support and is subjected to a stretching force, F, by attaching a mass, m, to its free end. Its length increases by Δl. This force is balanced by the upward elastic force due to the spring called the restoring force. To a good approximation, for small elongations, the force exerted by the spring is proportional to the amount of stretch:

$$F = -k\Delta l, \qquad\qquad (2)$$

where k is called the spring constant. Eq. (2) expresses Hooke's law. If the mass is now pulled down a small distance from the equilibrium position and then released, the restoring force causes the mass to oscillate up and down. Hooke's law gives the restoring force to be

$$F(t) = -ky(t) = -ky_m \cos(\omega t + \Phi) \qquad\qquad (3)$$

where $y(t)$ is the distance of the mass from its equilibrium position.

$$F(t) = ma(t)$$

$$\therefore \ a(t) = -\frac{k}{m} y_m \cos(\omega t + \Phi) \qquad\qquad (4)$$

We can write for the period of the SHM:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}. \qquad\qquad (5)$$

The period of oscillation for simple harmonic motion for a mass–spring system depends on the mass and the spring constant of the spring. In your experiment, the mass will include not only the mass added to the spring (m_p plus additional mass, m) but also the effective mass of the spring itself which is some fraction f of the spring mass m_s. The period is given by
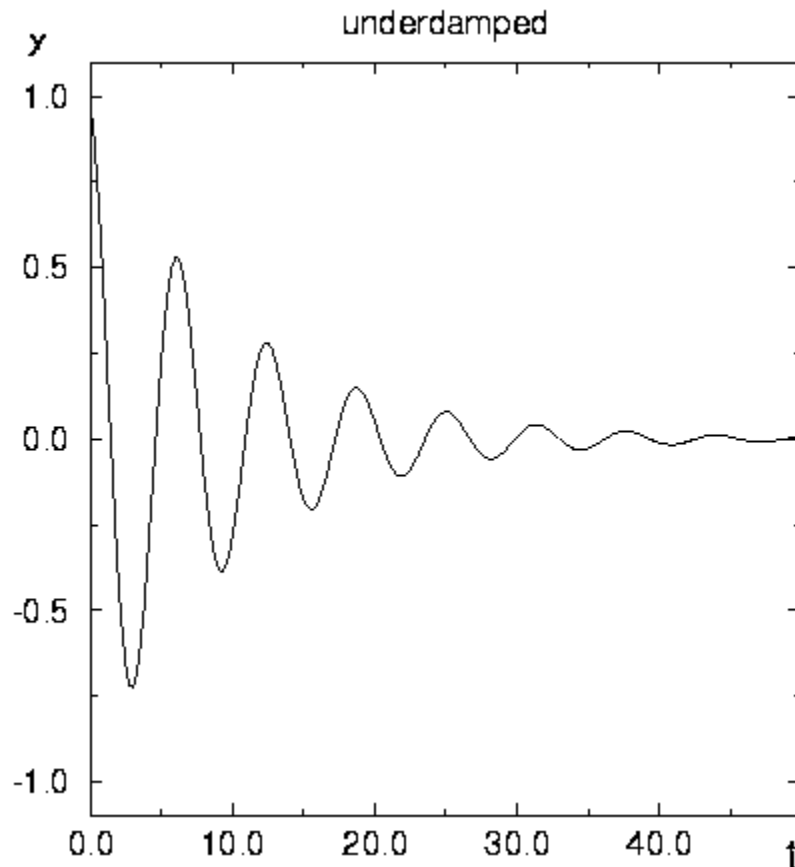
$$T = 2\pi\sqrt{\frac{m+m_p+fm_s}{k}} \qquad\qquad (6)$$

For uniform springs, the theoretical value of f is 1/3.
The motion is damped and the amplitude decreases with time, therefore

$$y(t)_m = e^{-(\beta/2)t}[y_m cos(\omega t + \Phi)] \qquad (7)$$

where β is the damping constant.

An example is given in the following figure.

Under-damped simple harmonic motion

2- Experiment

2-1 Object: To study Hooke's law, and simple harmonic motion of a mass oscillating on a spring.

2-2 Apparatus: Rotary motion sensor, thin string, uniform spring, balance, weight hanger and weight, computer Pasco Model 700 Interface, printer.

2-3 Procedure:

1- Adjust the apparatus. Setup the apparatus as shown in the figure. The 500 g weight on the right pan goes on the floor that the spring is attached to and on the other pan (that is on left) you put 40 g weight, m_p, so that the spring and the scale are vertical. Take that position of the pan as the equilibrium position.

2- Start the data collection – add extra mass m such as 10g, 20g, 30g and 40g to further stretch the spring. For each extra mass m, allow the pan to go down very slowly to the maximum position x and stop the data collection. Enlarge the position's table –you can record the extra stretch x. Repeat this with the different weights. Plot mg vs. x, draw a straight line through the origin and all the points. (g is the gravity acceleration constant). The slope of the line is the spring constant k

3- Remove the extra masses and come back to the equilibrium position (with 40g). Displace the whole weight by a small vertical distance (an inch or two). Release the system and start the collection data at the same time. When the oscillations (vibrations) vanish, stop the data collection.
On the graph of the position vs. time, use the sine fit to fit all sinusoidal plots. You can read the period of oscillations and calculate the value of the corresponding frequency ω_1 . Now, using the formula $\omega_1^2 = \dfrac{k}{m_1}$ calculate the real mass m_1 .What is your conclusion?

4- Use the computer pencil to draw the line that connects the top peaks of the position's waves.

5- Use the Natural Exponent fit to fit that line.
The exponent C gives you the value of $\beta/2$. Infer the value of the damping constant β .

6- Repeat the same experiment for the extra mass of 40 more grams.

7- Print all graphs and tables.

Simple Harmonic Motion