

Archimedes' Principle

1. Theory

We are aware that some objects float on some fluids, submerged to differing extents: ice cubes float in water almost completely submerged, while corks float almost completely on the surface. Even the objects that sink appear to weigh less when they are submerged in the fluid than when they are not. These effects are due to the existence of an upward 'buoyant force' that will act on the submerged object. This force is caused by the pressure in the fluid being increased with depth below the surface, so that the pressure near the bottom of the object is greater than the pressure near the top. The difference of these pressures results in the effective 'buoyant force', which is described by the Archimedes' principle.

According to this principle, the buoyant force F_B on an object completely or partially submerged in a fluid is equal to the weight of the fluid that the (submerged part of the) object displaces:

$$F_B = m_f g = \rho_f V g . \quad (1)$$

where ρ_f is the density of the fluid, m_f and V are respectively mass and the volume of the displaced fluid (which is equal to the volume of the submerged part of the object) and g is the gravitational acceleration constant.

2. Experiment

Object: Use Archimedes' principle to measure the densities of a given solid and a provided liquid.

Apparatus: Solid (metal) and liquid (oil) samples, beaker, thread, balance, weights, stand, micrometer, calipers, PASCO Science Workshop Interface, Force Sensors and a Macintosh Computer.

Part a.: A solid object of volume V , heavier than water. When completely submerged in water, the buoyant force is

$$F_B = \rho_w V g , \quad (2)$$

where ρ_w is the density of water. The mass of the object, M_a , when weighed in air is given by

$$M_a = \rho_s V , \quad (3)$$

where ρ_s is the density of the solid. The apparent weight of the object, $M_w g$, when immersed in water is the difference between its weight in air and the buoyant force:

$$M_w g = M_a g - \rho_w V g . \quad (4)$$

Therefore,

$$M_w = M_a - \rho_w V = M_a - \rho_w \frac{M_a}{\rho_s} , \quad (5)$$

$$\rho_s = \frac{M_a \rho_w}{(M_a - M_w)} . \quad (6)$$

The density of water ρ_w is taken as $1.0 \times 10^3 \text{ kg/m}^3$.

Part b.: Now for a liquid of unknown density ρ_l : If instead of water, an unknown liquid is used and the mass of the object of density ρ_s completely submerged in the liquid is represented by M_l , then

$$\rho_s = \frac{M_a \rho_l}{(M_a - M_l)} . \quad (7)$$

From Eqs. (6) and (7) we can obtain the density of the unknown liquid

$$\rho_l = \frac{(M_a - M_l) \rho_w}{(M_a - M_w)} . \quad (8)$$

Procedure:

1. Take a solid sample and measure its dimensions and then calculate the volume V . Mount the force sensor and plug it into the analog input of the Science Workshop Interface. Calibrate the force sensor using instructor's help. Weigh the sample in air, with a balance, and determine mass M_a .
2. Suspend the same solid object from the force sensor, and weigh it when it is fully immersed in water.
3. Now weigh the same object when it is fully immersed in an unknown liquid.

Calculation: Using the computer spreadsheet perform the following calculations.

1. Calculate the density of the solid from data from procedure (1) using $\rho_s = M_a/V$.
2. Use equation (6) and data from procedure (2) to compute the density of the solid object. Compare your answer with the result from task (1) and with standard values.
3. Use equation (7) to calculate the density ρ_l of the of the unknown liquid, and compare it with the value to be provided by your instructor.
4. Calculate the percent of error for each of your measurement.