The Howard University
Department of Physics and Astronomy

Master of Science Comprehensive and Doctor of Philosophy Qualifying Exam

Part 1: Classical Physics

August 22, 2000

Work out the solutions to *four* problems,– one from each group. Circle the numbers below to indicate your choice of problems.

1	2 3	4 5	6
Group A	Group B	Group C	Group D

1. Write in your code-letter here: ☐ .
2. Write your code-letter and a page number (in sequential order) on the *top right-hand* corner of each submitted answer sheet.
3. Write only on one side of the answer sheets.
4. Start each problem on a new answer sheet.
5. Stack your answer sheets by problem and page number, and then staple them (at the top *left-hand corner*) with this cover sheet on the top.

Good Luck!

Group A

1. An electron and a photon have the same total relativistic energy, E.

[7pts] *a.* Which one has the longer wavelength? Show your work. That is, solve for λ as a function of E for both particles.

[7pts] *b.* Does the relation you just derived hold for all energies, E? If not, what are the restrictions on the allowed values of E? Show your work.

[4pts] *c.* If $E = 1\,\text{MeV}$, what are the wavelengths of the two particle? What is the frequency of the photon?

[7pts] *d.* Consider a second electron that has the same wavelength as the photon above. What is the kinetic energy of this second electron? Show your work.

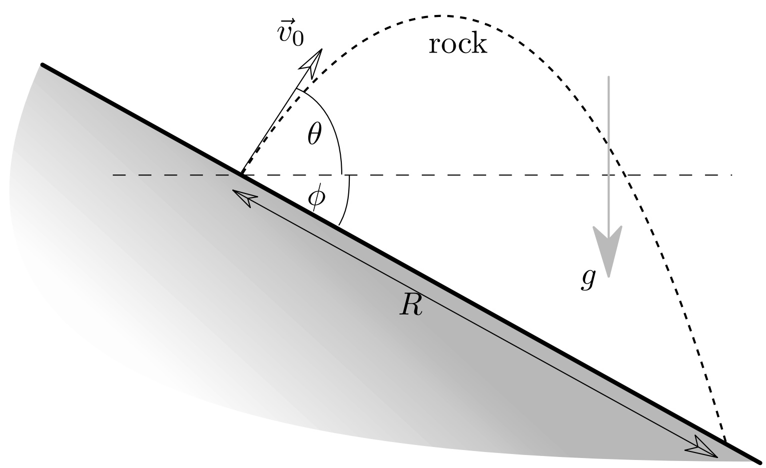
Group B

2. You are standing on the side of a hill which slopes uniformly downward, at a constant angle, ϕ (see figure below). You throw a rock at an angle θ with respect to the horizontal.

[10pts] *a.* Write down the Lagrangian and the equations of motion describing the motion of the rock, subject only to the uniform gravitational field and the initial velocity.

[10pts] *b.* Determine the optimal angle, θ_0, which maximizes the range, x, down the hill.

[5pts] *c.* Include now air resistance, by treating it as a frictional force, proportional to the speed of the rock (assuming it is spherical). Determine the necessary change(s) in the Lagrangian and the equations of motion.



3. A spring has the mass m and the spring constant k.

[10pts] *a.* When a bead of mass M is attached to the spring and set into small oscillation, show that the period is given by $T = 2\pi\sqrt{\frac{M+(m/3)}{k}}$.

[5pts] *b.* If the bead and the spring are set into motion from equilibrium by giving the bead the initial velocity of v_0 downward, calculate the amplitude of the induced oscillations.

As the bead and the spring are oscillating, their connection is cut when the bead is at the lowest point of its oscillation.

[5pts] *c.* Calculate the time it takes, from the time of cutting, for the bead to drop to a depth of h below the equilibrium point.

[5pts] *d.* Calculate the frequency of oscillation of the now bead-less spring.

Group C

4. A ball drops to the floor and bounces through inelastic collisions with the floor, eventually coming to rest. The speed just after each collision is α $(0 < \alpha < 1)$ times the speed just before the collision.

[10pts] *a.* If the ball is dropped from an initial height h, find the 'relaxation time' T_r, *i.e.*, the time it takes for the ball to come to rest on the floor.

Assume now that the ball carries a constant electric charge, q.

[5pts] *b.* How will the charge of the ball affect *qualitatively* its bouncing, and so the relaxation time, T_r?

[5pts] *c.* Derive the differential equation for the velocity of the bouncing ball before the first collision with the floor. To this end, you may consider law of the conservation of *total* energy, including the radiation of an accelerating particle. (Do not assume that the velocity or acceleration are constant!)

[5pts] *d.* Prove that the acceleration of the ball can no longer be constant.

5. A dielectric sphere of radius a and dielectric constant κ is placed in a uniform electric field $\vec{E}_0$, in vacuum.

[7pts] *a.* Determine the general form of the electrostatic potential inside, Φ_i, and outside, Φ_o, the sphere.

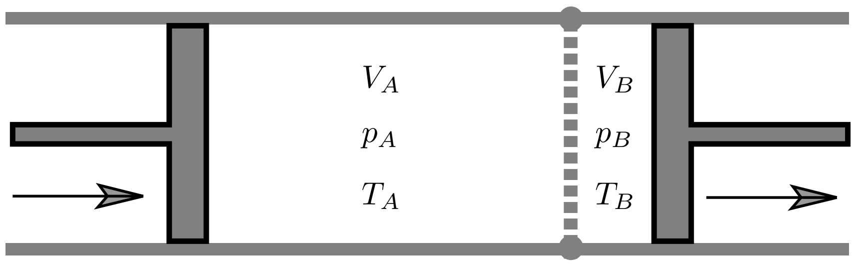
[7pts] *b.* State the boundary conditions that relate Φ_i and Φ_o.

[7pts] *c.* Use the boundary conditions to eliminate constants from the general solutions Φ_i and Φ_o.

[4pts] *d.* Determine the electric field $\vec{E}$ inside and outside the sphere.

Group D

6. Consider a gas which undergoes an adiabatic expansion (throttling process) from region of constant pressure p_A and volume V_A (initially $= V_{0A}$), to a region with constant pressure p_B and volume V_B (initially $= 0$). The regions are separated by a porous wall, as indicated on the figure below.



[7pts] *a.* By considering the work done by the gas in the process, show that the initial and final enthalpies of the gas are equal.

[4pts] *b.* Does this conclusion apply to intermediate states? Why?

[7pts] *c.* Show that when enthalpy is conserved in a process (such as in the throttling process above), the change in the temperature, $\triangle T$, is for small pressure difference, $\triangle p$, is given by

$$\triangle T = \triangle p \left(\frac{V}{c_p}\right)(\alpha T - 1) \ ,$$

where

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p \ , \quad \text{and} \quad c_p = \left(\frac{\partial U}{\partial T}\right)_p \ .$$

[7pts] *d.* Using the above result, discuss the possibility of using the process to cool an ideal gas, and also a more realistic gas for which $p(V-b) = RT$. Explain your results.

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Part 2: Modern Physics

August 25, 2000

Work *four* problems, at least one from each group. Circle the numbers below to indicate your chosen problems.

1 2	3 4	5 6
Group A	Group B	Group C

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Good Luck!

Group A

1. A system of two-state atoms is in a thermal radiation field of radiation density

$$\rho(\nu) \;=\; \frac{8\pi h\nu^3}{c^3}\,\frac{1}{e^{h\nu/kT}-1}\;,$$

at temperature T. The populations N_1, in (the lower energy) state 1, and N_2, in (the higher energy) state 2, are in thermal equilibrium. The following three processes take place:

(1) Atoms can be excited from state 1 to state 2 by absorption of a photon according to

$$\left(\frac{\mathrm{d}N_1}{\mathrm{d}t}\right)_{\mathrm{ab}} = -B_{12}\,N_1\,\rho(\nu)\ .$$

(2) Atoms can decay from state 2 to state 1 by spontaneous emission of a photon according to

$$\left(\frac{\mathrm{d}N_2}{\mathrm{d}t}\right)_{\mathrm{sp}} = -A_{21}\,N_2\ .$$

(3) Atoms can decay from state 2 to state 1 by stimulated emission of a photon according to

$$\left(\frac{\mathrm{d}N_2}{\mathrm{d}t}\right)_{\mathrm{st}} = -B_{21}\,N_2\,\rho(\nu)\ .$$

[3pts] *a.* Write down the equilibrium relation between $\left(\frac{\mathrm{d}N_1}{\mathrm{d}t}\right)_{\mathrm{ab}}$, $\left(\frac{\mathrm{d}N_2}{\mathrm{d}t}\right)_{\mathrm{sp}}$ and $\left(\frac{\mathrm{d}N_2}{\mathrm{d}t}\right)_{\mathrm{st}}$.

[5pts] *b.* Determine the ratio N_2/N_1.

[12pts] *c.* Calculate the ratio of coefficients A_{21}/B_{21} and B_{21}/B_{12}.

[3pts] *d.* Suppose you want to make a short-wavelength laser. From the ratio of stimulated to spontaneous emission, determine how does the power of the pump scale with the wavelength.

[2pts] *e.* For a maser (with the radiation wavelength in the centimeters' regime), estimate the (order of magnitude of the) energy gap between the two levels.

Group A

2. Consider a heteronuclear diatomic molecule with the moment of inertia I, which is constrained to rotate in a plane.

[5pts] *a.* Show that the energy levels are given by $E_j = \frac{\hbar^2}{2I} j(j+1)$.

[5pts] *b.* Determine the allowed values of j, and the degeneracy of the corresponding energy levels.

[5pts] *c.* Using quantum statistical mechanics, find expressions for the partition function Z and the average energy $\langle E \rangle$ of this system. (You need not evaluate these expressions.)

[5pts] *d.* By simplifying the expressions in part *c.*, derive an expression for the specific heat, $C(T)$, that is valid at very low temperatures. (Hint: Include in your calculation only the two lowest-energy states.)

[5pts] *e.* By simplifying the expressions in part *c.*, derive an expression for the specific heat, $C(T)$, that is valid at very high temperatures. (Hint: Approximate the discrete sum by an integral.)

Group B

3. A D_2 molecule, at T=30 K and t=0, is known to be in the state

$$\psi(r,\theta,\phi,0) = \frac{R(r)}{\sqrt{26}}\Big[3Y_1^1(\theta,\phi) + 4Y_7^3(\theta,\phi) + Y_7^1(\theta,\phi)\Big]\ ,$$

where $R(r)$ is a fixed (and normalized) radial function and $Y_\ell^m(\theta,\phi)$ are the usual spherical harmonics.

[10pts] *a.* Calculate the measured values of L and L_z in this state, and the probabilities with which these values will occur.

[8pts] *b.* Calculate $\psi(r,\theta,\phi,t)$, at $t>0$.

[7pts] *c.* Calculate $\langle E\rangle$ (in eV) for the molecule, at $t>0$. (Use $\hbar/4\pi Ic = 30.4\,\text{cm}^{-1}$.)

4. The wave-function of a particle is known to be

$$\psi(x) = \begin{cases} A\big[\cos\left(\frac{\pi x}{a}\right)+1\big] & \text{for } |x|<a, \\ 0 & \text{for } |x|>a. \end{cases}$$

[2pts] *a.* Sketch $\psi(x)$.

[3pts] *b.* Determine the normalization constant, A.

[10pts] *c.* Evaluate $\triangle x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$ and $\triangle p_x = \sqrt{\langle p_x^2\rangle - \langle p_x\rangle^2}$.

[5pts] *d.* Calculate the probability that the particle is found within $|x|<\frac{a}{2}$.

[5pts] *d.* For the state to be interpreted in terms of a classical particle, specify the region of validity of this interpretation. (Hint: the kinetic energy of a classical particle is non-negative. However, identify the expression for the kinetic energy very carefully!)

Group C

5. Consider a system of two *distinguishable* particles, labeled 1 and 2, with total angular momentum quantum numbers j_1=1 and j_2=2.

[5pts] *a.* Using the 'product basis,' $|j_1, m_1\rangle\,|j_2, m_2\rangle$, list all the distinct states and state their total number.

[7pts] *b.* Determine the possible values of the total angular momentum, j, of the 2-particle system. State the number of 'projections,' m, that each value of j may have.

[8pts] *c.* Write the j=2 and m=1 'composite' state, $|2, 1\rangle$, in terms of the 'product basis'. (Use symmetry arguments, if possible, and explain your reasoning.)

[5pts] *d.* Using that $|2, 2\rangle = |1, 1\rangle\,|1, 1\rangle$, determine the correctly normalized expression for the state $|2, 1\rangle$ in terms of the 'product basis.' Recall that $\vec{\mathsf{J}} = \vec{\mathsf{J}}_1 + \vec{\mathsf{J}}_2$, and that

$$\mathsf{J}_- \,|j, m\rangle = \sqrt{j(j+1) - m(m-1)}\,|j, m-1\rangle \ .$$

6. A harmonic oscillator of charge q has the Hamiltonian

$$H_0 = \tfrac{1}{2m}\,p^2 + \tfrac{1}{2}m\omega^2\,x^2 \ .$$

The ground-state wave-function may be written as

$$\psi_0(x) = \sqrt[4]{\alpha^2/\pi}\ \exp\big(-\tfrac{1}{2}\alpha^2 x^2\big) \ .$$

[5pts] *a.* Determine the constant α.

[5pts] *b.* Determine the Ground state energy, E_0.

The oscillator is being perturbed by the application of a weak electric field $\vec{E}$ in the positive x direction, so that the potential energy becomes

$$V(x) = \tfrac{1}{2}m\omega^2\,x^2 - q|\vec{E}|x \ .$$

[5pts] *c.* Show that the energy of the ground state remains unchanged to within first order in stationary state perturbation theory.

[5pts] *d.* Solve the perturbed Schrödinger equation *exactly*, and find the *exact* energy and *exact* wave-function of the ground state. (Hint: complete the square in the perturbed potential, $V(x)$.)

[5pts] *e.* Calculate the energy of the ground state to within second order in stationary state perturbation theory, and compare with the exact result from part *d*.