

The Howard University Department of Physics and Astronomy

Master of Science Comprehensive and Doctor of Philosophy Qualifying Exam

Part 1: Classical Physics

August 29, 2006

Work *four* problems, at least one from each group. Circle the numbers below to indicate your chosen problems.

1 **2 3** **4 5 6**
Group A Group B Group C

1. Write in your code-letter here: .
2. Write your code-letter and a page number (in sequential order) on the *top right-hand* corner of each submitted answer sheet.
3. Write only on one side of the answer sheets.
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Good Luck!

Group A

1. One mole of a classical ideal gas undergoes the cycle in the PV plane shown in the figure; 1-2 is an adiabatic process; 2-3 is an isothermal process; 3-1 is an isobaric process; and $V_3 = 2V_1$.

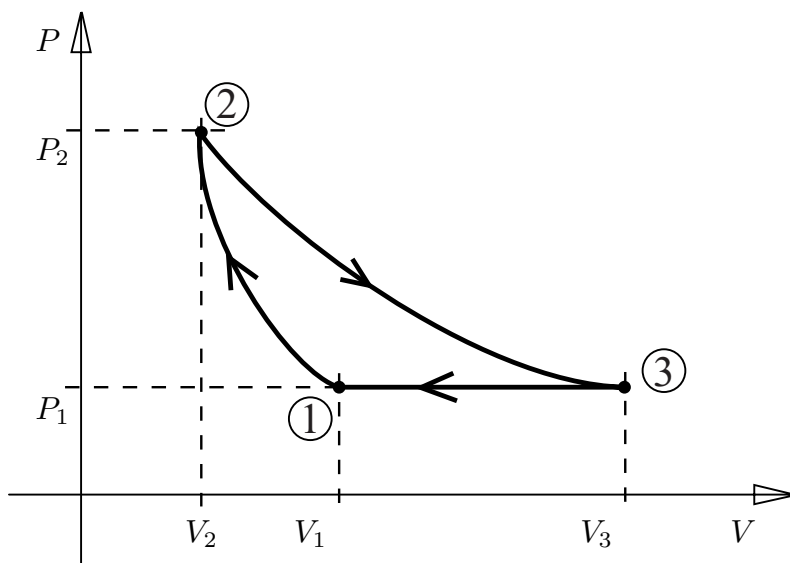
[10pts] a. Show that the equation for an adiabatic process can be written as

$$pV^\gamma = \text{constant}$$

where $\gamma = C_P/C_V$, and C_P and C_V are the molar specific heats at constant pressure and volume.

[10pts] b. In terms of P_1 , V_1 , the ratio of specific heats γ , and the gas constant R , what is the heat absorbed by the gas and the work done by the gas in each of the three steps 1-2, 2-3, and 3-1? What is the internal energy of the gas at each of the three states 1, 2, and 3?

[5pts] c. Compute the efficiency of a reversible engine utilizing this cycle. Compare this efficiency with the efficiency of a Carnot engine working between the same extremes of temperature.



Group B

2. A rocket is propelled in a forward direction by ejecting mass in a backward direction in the form of gases resulting from the combustion of fuel. Let m be the mass of the rocket (including fuel) at a given time t and let the velocity of the rocket relative to the ground be \vec{v} . Let the velocity of the exhaust gases from the rocket be \vec{u} with respect to the rocket.

[10pts] a. Show that the rate of change of momentum for the system can be given by

$$\frac{d\vec{p}}{dt} \approx m \frac{d\vec{v}}{dt} - \vec{u} \frac{dm}{dt},$$

where $dm/dt < 0$.

[10pts] b. The rate of change of the momentum, $d\vec{p}/dt$, is equal to the applied external force, \vec{F} . The force \vec{F} may be a gravitational force, the force of air resistance or any other external force. Consider the case for $\vec{F} = 0$. (This may be the case when the rocket is in outer space, far from any stars or planets.) Show that if the rocket initially has a speed \vec{v}_0 and a mass m_0 , then the velocity at some later time is given by

$$\vec{v} = \vec{v}_0 - \vec{u} \ln \left(\frac{m_0}{m} \right)$$

[5pts] c. Assume that near the Earth's surface the only external force is that given by the Earth's gravitational attraction. If the rocket starts from rest on its launch pad, what is the final speed of the rocket at time t ? (Assume that the acceleration due to gravity is constant over the range of interest.)

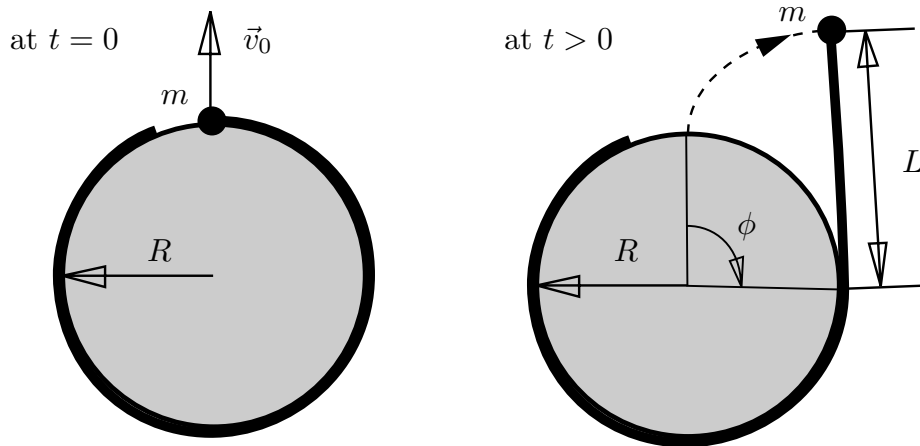
Group B

3. A point mass m under no external forces is attached to a weightless cord fixed to a cylinder of radius R . Initially the cord is completely wound up so that the mass touches the cylinder. At time $t = 0$, a radially-directed impulse is given to the mass so that its initial velocity is v_0 , and the cord starts unwinding. At time t , the length of cord unwound is L , the velocity of the mass is v , and ϕ is the angle through which the cord has unwound.

[10pts] a. Find the equation of motion in terms of some suitable generalized coordinate.

[10pts] b. Find the general solution satisfying the initial condition.

[5pts] c. Find the angular momentum of the mass about the cylinder axis, using the result of part (b).



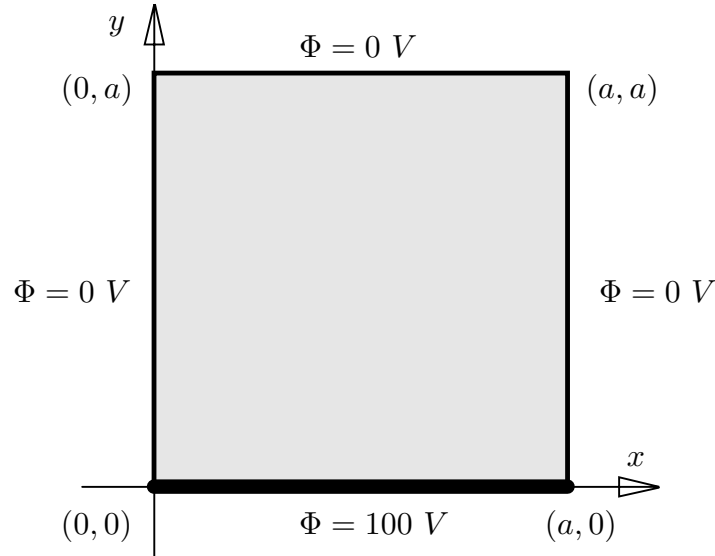
Group C

4. Consider a square, with sides of length a , where the electric potential Φ is 100 V on one edge of the square (i.e., along the side where $y = 0$), and $\Phi = 0$ V on the other three edges.

[15pts] a. What is the general solution for $\Phi(x, y)$ inside the square?

[10pts] b. What is the numerical value of Φ at the center of the square, that is, at the position $(\frac{1}{2}a, \frac{1}{2}a)$?

Hint: In two dimensional problems the system is considered to be very long in the z direction compared to its length in the x or y directions. Hence, we may use the approximation that Φ is independent of z , i.e., Φ is a function of x and y only.



Group C

5. A current I flows in a circular loop of radius R in the xy plane, with the center of the circle at the origin.

- [5pts] a. Find the magnitude and direction of the magnetic field B at the point $z = \rho$ on the z axis?

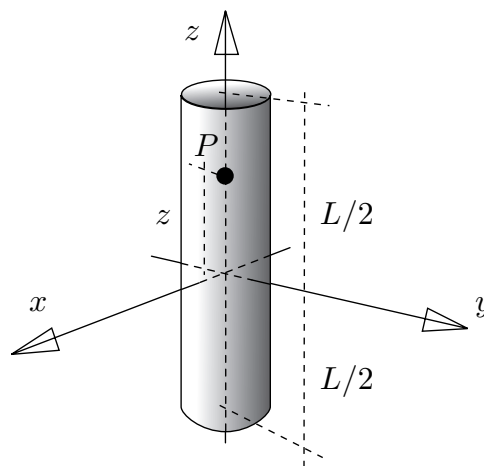
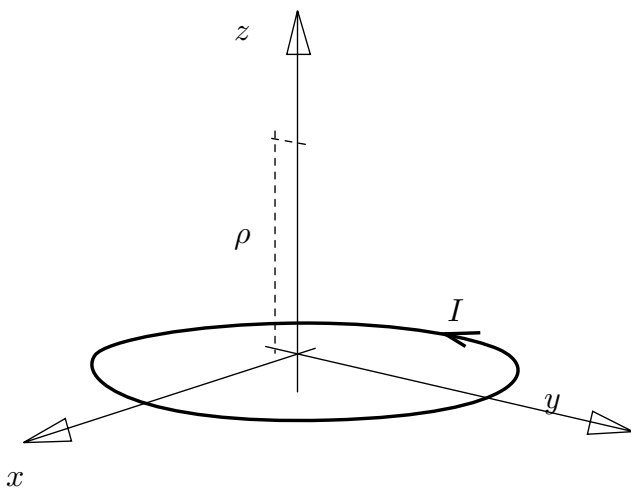
A solenoid has N turns of wire evenly and closely wound on a plastic tube of length L and radius R ($L \gg R$) and carries a current I . The axis of the solenoid is the z axis, and the center of the solenoid is at the origin.

- [10pts] b. What is the magnetic field B at point P , a distance z from the origin along the z axis?

- [5pts] c. Plot B versus z . What are the values of B at $z = 0$, $z = L/2$, and $z \gg L/2$?

- [5pts] d. For $L \gg R$, how does your result compare with the result from Ampere's law?

Hint: The law of Biot-Savart is useful.



Group C

6. In the absence of dielectric and magnetic materials (i.e., free space),

[2pts] a. Write down Maxwell's equations in their integral form.

[3pts] b. If magnetic monopoles existed, which of Maxwell's equations would be altered and what would be their new form?

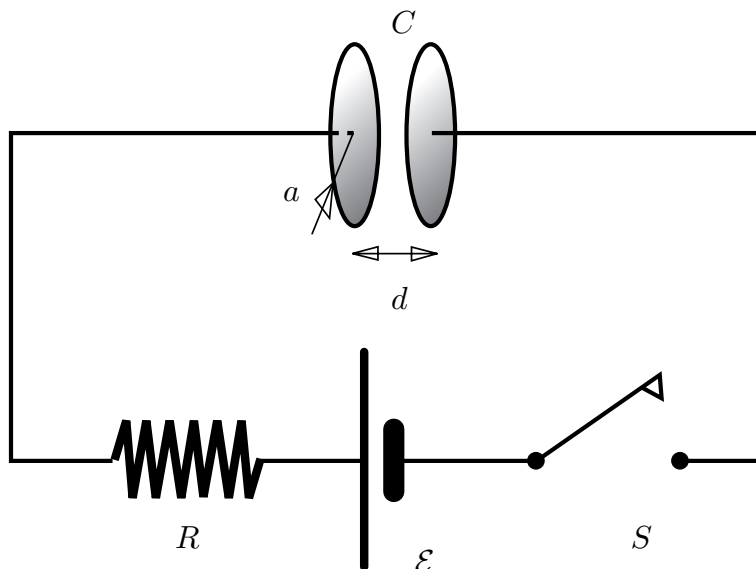
Suppose an air gap capacitor with circular plates (capacitance = C , plate separation = d , radius = a) is charged by a battery (EMF = \mathcal{E}) through a resistor (resistance = R). See the figure. The capacitor is initially uncharged and the switch S is closed at time $t = 0$. (Hint: You can ignore the fringing of the electric field \vec{E} .)

[5pts] c. Determine the charge on the capacitor as a function of time.

[5pts] d. Determine the rate of change of the electric field between the plates as a function of time.

[5pts] e. Find the magnetic field induced between the plates as a function of time for $r \leq a$ and $r \geq a$.

[5pts] f. By integrating the Poynting vector \vec{S} over the cylindrical boundary of the capacitor gap, show that the rate at which energy enters the capacitor equals the rate at which electrostatic energy is being stored in the electric field of the capacitor.



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Part 2: Modern Physics

August 31, 2006

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Group C

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Group A

1. A radioactive nucleus of type A decays into a nucleus of type B with decay constant λ_A . Nucleus B is also radioactive; it decays into a nucleus of type C and has a decay constant λ_B . Nucleus C is stable; it does not radioactively decay. The quantity λ is the probability that a particular nucleus will decay in any unit interval of time. Initially, the number of nuclei of type A is N_{A0} , and there are no nuclei of types B or C .

- [5pts] a. Show that at time t the number of nuclei of type A is given by

$$N_A(t) = N_{A0}e^{-\lambda_A t}.$$

- [10pts] b. Obtain an expression for

$$\frac{d}{dt}N_B(t),$$

the time rate of change of the number of nuclei of type B at time t .

- [5pts] c. Solve the equation obtained in part (b) for $N_B(t)$, and show that the solution is

$$N_B(t) = \left(\frac{\lambda_A}{\lambda_B - \lambda_A} \right) N_{A0} \{ e^{-\lambda_A t} - e^{-\lambda_B t} \}.$$

- [5pts] d. Find the number of nuclei of type C at time t , $N_C(t)$.

Group A

2. A photon and an electron collide head-on with relativistic energies \mathcal{E} and E , respectively, as shown in the figure below. (All motion is one dimensional.) After the collision, their energies are \mathcal{E}' and E' .

- [5pts] a. Show that for the incident electron

$$E = \sqrt{p^2 c^2 + m^2 c^4},$$

where m and p are the electron rest mass and relativistic momentum, respectively.

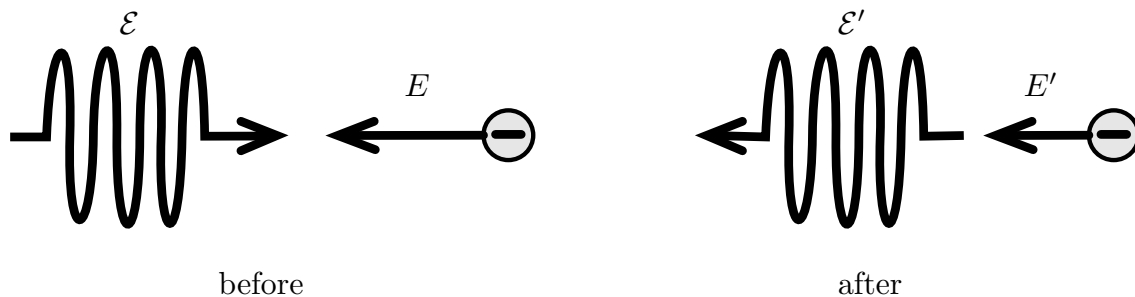
- [10pts] b. Show that for the backward Compton-scattered photon, the energy \mathcal{E}' is given by

$$\mathcal{E}' = \frac{E + \sqrt{E^2 - m^2 c^4}}{E - \sqrt{E^2 - m^2 c^4} + 2\mathcal{E}}.$$

Simplify this expression for \mathcal{E}' for the case when $E \gg mc^2$.

- [5pts] c. Find the wavelength of the scattered photon if a 700-nm photon collides with a 10-GeV electron?

- [5pts] d. Show that a free electron (say at rest) cannot absorb a photon. In other words, the free electron must scatter the photon.



Group B

3. For a harmonic oscillator with particle mass m and force constant k , Schrodinger's equation is

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + \frac{1}{2}kx^2\psi = E\psi.$$

- [5pts] a. Reduce Schrodinger's equation to the simple form

$$H\psi = -\frac{1}{2} (D^2 - y^2) \psi = \mathcal{E}\psi$$

by using the transformation of variables $x = \alpha y$ and $E = \beta\mathcal{E}$ (D is the differential operator d/dy). What are α and β and what is the rationale for choosing them?

- [5pts] b. Let $a = D + y$ and $b = D - y$. Obtain an expression for the Hamiltonian in terms of a and b only.
- [5pts] c. Consider the commutators $[H, a]$ and $[H, b]$. Show that a and b can be identified with the raising and lowering operators when acting on eigenstates of H .
- [5pts] d. Calculate the ground state wave function ψ_0 by requiring that the lowering operator acting on ψ_0 yields zero (there is no need to normalize the wave function). Determine the result of H acting on this ground state, $(H\psi_0)$.
- [5pts] e. Calculate the wave functions, $\psi_1 \propto (a\psi_0)$ and $\psi_2 \propto (a^2\psi_0)$, of the first and second excited states (again, there is no need for normalization). Determine the result of H acting on these states, $(H\psi_1)$ and $(H\psi_2)$.

Group B

4. Consider the vector angular momentum operator $\vec{L} = \vec{r} \times \vec{p}$.

[5pts] a. Prove the commutation relation $[L_x, L_y] = i\hbar L_z$,

[5pts] b. Explain why the commutation relations $[L_y, L_z] = i\hbar L_x$ and $[L_x, L_z] = -i\hbar L_y$ follow trivially from the result in part (a).

[5pts] c. Show that $[L^2, \vec{L}] = 0$.

The results so far imply that we can find simultaneous eigenfunctions ψ of the operators L^2 and L_z with eigenvalues λ and μ respectively

$$L^2\psi = \lambda\psi,$$

$$L_z\psi = \mu\psi.$$

[5pts] d. Show that the wave functions $(L_+\psi)$ and $(L_-\psi)$ are eigenfunctions of L^2 , where L_+ and L_- are the raising and lowering operators. Find their L^2 -eigenvalues.

[5pts] e. Show that these same two wave functions are eigenfunctions of L_z . Find their L_z -eigenvalues.

(A possibly useful relation is $[x, p_x] = i\hbar$.)

Group C

5. Consider an electron (of mass m_e) trapped but otherwise moving freely within a horizontal $L \times 2L$ rectangle such that $x \in [0, L]$ and $y \in [0, 2L]$.

- [3pts] a. Write down the Hamiltonian and the boundary conditions for this electron.
- [3pts] b. Determine the wave-functions, $|n_x, n_y\rangle$, and energies, E_{n_x, n_y} , for this electron.
- [2pts] c. Find the most probable locations for the electron in the state $|2, 2\rangle$.
- [4pts] d. Normalize the wave-functions for this electron.
- [6pts] e. Specify the degeneracy for those states where

$$E_{n_x, n_y} \leq 37 \frac{\pi^2 \hbar^2}{8m_e L^2}.$$

- [7pts] f. For an electron with the normalized wave-function

$$|\psi\rangle \equiv \frac{\sqrt{30}}{4L^3} \sin\left(\frac{\pi x}{L}\right) y(2L-y),$$

find the probability P_{n_x, n_y} that it is in the state $|n_x, n_y\rangle$.

Useful integral: $\int_0^\pi d\phi \sin(n\phi) \sin(m\phi) = \frac{\pi}{2} \delta_{n,m}$.

Group C

6. Consider the effects of a weak, homogeneous and constant magnetic field \vec{B} on the Hydrogen atom, neglecting spin and relativistic corrections.
- [6pts] a. Specify the degeneracy among the fourteen lowest-energy states ($n = 1, 2, 3$), the contribution of this magnetic field to the H-atom Hamiltonian, and the quantum numbers, (n, ℓ, m) , of the states that are affected by this magnetic field.
- [10pts] b. Calculate the shift in the energies of all the -3.4eV states, to 1st and 2nd order in perturbation theory and specify the resolution of degeneracies, if any.
- [9pts] c. Calculate the shift in the energies of all the -1.5eV states, and specify the resolution of degeneracies, if any.

Hint: $1\text{R}=13.6\text{eV}$ is the characteristic energy of the Hydrogen atom. You may want to first calculate the general matrix element, $\langle n', \ell', m' | H_B | n, \ell, m \rangle$ with H_B the appropriate perturbation Hamiltonian due to \vec{B} , and then the specific quantities for parts *b* and *c*.