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—Classical Mechanics—

1. The system of a particle of mass m acted on by an inverse square force, with potential $V(r) = +k/r$, has a Lagrangian

$$L = \frac{m}{2}\left[\, \dot{r}^2 + r^2\dot{\theta}^2 \,\right] - \frac{k}{r}$$

in polar coordinates (r, θ).

a. Write expressions for the canonical momenta.

b. Construct the Hamiltonian H of the system.

c. Obtain Hamilton's equations of motion.

d. Is H a constant of motion? Explain.

2. A thin hoop of radius R and mass M lies and oscillates in the vertical plane with one point of the hoop fixed, as shown on the figure below. Attached to the hoop is a point mass M constrained to move without friction along the hoop. The system is in a uniform gravitational field with acceleration g. Consider only small oscillations.

a. Calculate the normal mode frequencies ω_1 and ω_2.

b. Find the normalized normal-mode eigenvectors.

c. Find the normal coordinates and show that they diagonalize the Lagrangian.

3. A particle of mass m moves under the action of a force the potential of which is given by $V(r) = kr^4$, where $k > 0$.

a. Calculate the force $f(r)$ and plot $V(r)$ *vs.* r and $f(r)$ *vs.* r.

b. Plot $V_{\text{eff}}(r)$ *vs.* r. Discuss the motion of the particle for energies: $E = V_0$, $E > V_0$ and $E < V_0$, where $V_0 = V_{\text{eff}}(r_0)$ is the minimal value of $V_{\text{eff}}(r)$. (Recall: V_{eff} includes, in addition to $V(r)$, also the "centrifugal potential" term.)

c. Find the values of angular momentum ℓ and energy E for the radius $r = r_0$ of a circular orbit.

d. Calculate the period T of circular motion with radius r_0.

4. Consider two coupled pendula as shown in the figure below.

a. Find the potential energy of this system.

For small oscillations about the equilibrium positions, find:

b. the components V_{ij} of the potential energy matrix, $\mathbf{V}$;

c. the components T_{ij} of the kinetic energy matrix, $\mathbf{T}$;

d. the normal mode frequencies;

e. the normal modes of oscillation;

f. the equations of motion;

g. the general solution.