HOWARD UNIVERSITY
WASHINGTON, DC, 20059

DEPARTMENT OF PHYSICS AND ASTRONOMY | 2355 Sixth Str., NW
(202)-806-6245 (Main Office) | (202)-806-5830 (FAX)

—Statistical Mechanics—

1. Consider a solid to be a collection of N non-interacting three-dimensional isotropic harmonic normal mode oscillators arranged on a regular cubic lattice with temperature T.

a. Considering that all the oscillators have the same vibrational frequency, calculate the specific heat of the solid assuming classical energy values for the oscillators.

b. Assuming (as Einstein did) that the normal mode oscillators have quantum energy levels with some vibrational constants, ω_E, obtain the specific heat for this case and demonstrate the limit required to obtain the result in part a.

c. Find the low temperature limit of the quantum heat capacity and explain why it is reasonable in terms of the model.

2. a. An ideal gas undergoes adiabatic expansion. Obtain the temperature as a function of pressure.

b. Impose the condition of gravitational equilibrium at a height h above the ground and obtain a differential relationship between $\mathrm{d}P$ and $\mathrm{d}h$ from the ideal gas law. (Assume h much smaller that the Earth's radius.)

c. A simple model of the atmosphere assumes that the temperature is independent of h. Find p as a function of h for this simple model (Boltzmann's relation).

d. A more realistic model neglects thermal conduction and assumes that as the mass of the air rises or descends, it undergoes and adiabatic reversible process. Obtain an expression for $\frac{\mathrm{d}T}{\mathrm{d}h}$ under this assumption.

e. Given $C_P/C_V = 1.4$ and $m = 0.029\,kg/\text{mole}$ for air, and $k = 1.381 \times 10^{-23} J/K$, $N_A = 6.022 \times 10^{23}$, $g = 9.80\,m/s$, obtain an estimate for the change of the temperature with altitude at $h = 0$, and for the change in pressure with altitude at $P = P_0 = 0$ and $h = 0$.

3. For an arbitrary system with energy levels E_n,

a. show that the average energy is

$$\overline{E} \quad = \quad -\frac{\partial \ln Q}{\partial \beta} \ ,$$

where $Q = \sum_n e^{-\beta E_n}$ is the partition function.

b. Express the dispersion of energy, $\sigma_E^2 = \overline{(\triangle E)^2}$ in terms of the heat capacity, C_V, and the temperature, T.

c. Calculate $\frac{\sigma_E}{\overline{E}}$ for and ideal monoatomic gas using the results obtained above.

4. A Schottky defect is a simple crustal defect in which an atom migrates to the surface of the crystal. Assume that an occupied lattice site has energy 0, while a defect has energy E.

a. What is the canonical partition function for the system?

b. Show that the average number of defects is

$$\overline{n} = \frac{N}{e^{\beta E} + 1} \ .$$

(Hint: Recall the binomial theorem $(a+b)^N = \sum_{n=0}^{N} \frac{N!}{n!\,(N-n)!} a^n b^{N-n}$.)