The Howard University
Department of Physics and Astronomy

Master of Science Comprehensive and Doctor pf Philosophy Qualifying Exam

Written exam: Classical Physics

August 17, 2010

Attempt to solve ***four*** problems, at least one from each group. Circle the numbers below to indicate the problems you chose to solve:

1 2	3 4 5	6 7 8
Group A	Group B	Group C

1. Write in your chosen code-letter here: [ ]
2. Write your code-letter and a page number (in sequential order) on the *top right-hand* corner of each submitted answer sheet.
3. Write only on one side of your answer sheets.
4. Start each problem on a new answer sheet.
5. Stack your answer sheets by problem and page number, and then staple them (at the top *left-hand corner*) with this cover sheet on the top.

Problem 1.

A particle of mass m moving in 3-dimensional space, subject to two forces: (**1**) a uniform gravitational field, $\vec{F}_g = -mg\hat{z}$, where the z-axis is vertical and increases upward, and $\hat{z}$ is the unit vector in this direction; and (**2**) a central force $\vec{F}_c = -\frac{A}{\sqrt{r}}\hat{r}$, directed towards a fixed point in space.

[5 pt] ***a.*** Write down the potential function U. (Although not necessary, it is convenient to select the origin of U so that $U = 0$ at the location of the mentioned fixed point in space.)

[5 pt] ***b.*** What is the minimum number of degrees of freedom in this problem? In the following parts, you may restrict yourself to this minimum number.

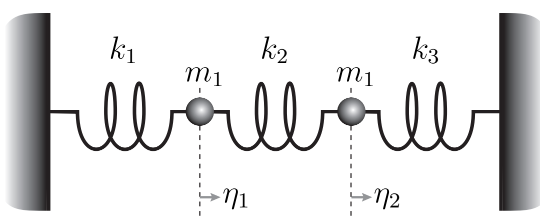
[5 pt] ***c.*** Write down the Lagrangian function for this problem.

[5 pt] ***d.*** Are there any conserved quantitites? Justify your answer.

[5 pt] ***e.*** Find the Euler-Lagrange equations of motion.

Problem 2.

Consider two beads of different masses connected by three springs as depicted below:



[4 pt] ***a.*** Find the kinetic and potential energy for the system i terms of the coordinates η_1, η_2.

[4 pt] ***b.*** Obtain the equations of motion from the Lagrangian.

[4 pt] ***c.*** For the case when $k_1 = k_2 = k_3 = k$ and $m_1 = m_2 = m$, find the characteristic frequencies.

[4 pt] ***d.*** Obtain the relationships between the amplitudes a_{11}, a_{12}, a_{21} and a_{22}.

[5 pt] ***e.*** Assuming that at $t = 0$ the masses are not moving and the second mass is displaced a distance A, write down the solution for the motion of the masses (the η's).

[4 pt] ***f.*** Write down the expression for the normal coordinates for the problem.

Problem 3.

From the Lagrangian for a charged particle in an electromagnetic field in Cartesian coordinates $L = \frac{1}{2}m\vec{v}^2 - q\Phi + \frac{q}{c}\vec{A}\cdot\vec{v}$,

[4 pt] ***a.*** compute the momenta,

[6 pt] ***b.*** compute an expression for the velocities $(\dot{x}, \dot{y}, \dot{z})$ in terms of the momenta found in part a.

[6 pt] ***c.*** From the Lagrangian, momenta and velocity expressions, compute the Hamiltonian.

[9 pt] ***d.*** From the Hamiltonian, compute the Lorentz force.

Problem 4.

A sphere of radius R has a total charge q, which is uniformly distributed throughout the volume of the sphere.

[4 pt] ***a.*** Compute the charge density within the sphere.

[6 pt] ***b.*** Compute the electrical field at points inside the sphere.

[6 pt] ***c.*** Compute the electrical field at points outside the sphere.

[9 pt] ***d.*** Sketch a graph showing the electrical field as a function of distance from the center of the sphere out to $4R$. That is, plot $|\vec{E}|$ *vs.* r from $r = 0$ to $r = 4R$.

Problem 5.

Identifying Faraday's law of induction, $\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$, as the differential form of one of Maxwell's equations,

[9 pt] ***a.*** write down the remaining three Maxwell's equations in the same system of units and in vacuum, but in the presence of charges and currents, and define each symbol in them.

[10 pt] ***b.*** For a polarized plane-wave propagating in the x-direction the $\vec{E}$-field of which oscillates in the y-direction, show that $E_y = cB_z$.

[6 pt] ***c.*** Compute the momentum vector (magnitude and direction) of this plane-wave.

Problem 6.

Consider N classical harmonic oscillators with coordinates and momenta (q_i, p_i), with the Hamiltonian given as

$$H(q_i, p_i) = \sum_{i=1}^{N} \left[\frac{p_i^2}{2m} + \tfrac{1}{2}, \omega^2 q_i^2 \right]$$

[8 pt] ***a.*** Calculate the entropy, S, as a function of the total energy, E.

[8 pt] ***b.*** Calculate the energy, E, and the heat capacity, C, as functions of temperature, T, and the number of oscillators, N.

[9 pt] ***c.*** Find the joint probability density, $P(p, q)$ for a single oscillator. Hence calculate the mean kinetic energy, and the mean potential energy for each oscillator.

Problem 7.

Check units: if F/m is energy, how come $F = E - TS$? It is found that when a 'HU-spring' is stretched to a certain length, it breaks. Before the spring breaks (*i.e.*, at small lengths), the free energy of the spring is given by $F/m = \frac{1}{2}kx^2$, where $F = E - TS$, m is the mass of the spring, x is its *length per unit mass*. After breaking (*i.e.*, at large lengths), $F/m = \frac{1}{2}h(x-x_0)^2 + c$. Throughout, k, h, x_0 and c are independent of x, but do depend on the temperature, T. Note that $k > h$ and $c, x_0 > 0$ for all T.

[5 pt] ***a.*** Determine the equations of state f = tension = $f(T, x)$, for the spring at small and long lengths.

[5 pt] ***b.*** Similarly, determine the chemical potentials, $u = \left(\frac{\partial F}{\partial M}\right)_{T,L}$ where L is the total length of the spring.

[5 pt] ***c.*** Show that $u = F/m - fx$.

[5 pt] ***d.*** Find the force that at a given temperature will break the spring.

[5 pt] ***e.*** Determine the discontinuous change in x when the spring breaks.

Problem 8.

An ideal gas of particles, each of mass m at temperature τ, is subjected to an external force stemming from the potential $U(x) = Ax^n$, with $0 \le x \le \infty$, $A > 0$ and $n > 0$.

[10 pt] ***a.*** Compute the average potential energy per particle.

[15 pt] ***b.*** Compute the average potential energy per particle in this gas if it is placed in a uniform gravitational field.

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Written exam: Modern Physics

August 19, 2010

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Problem 1.

Consider the graphical representation of objects moving with relativistic relative speeds.

[8 pt] ***a.*** Describe the so-called 'twin paradox' in special relativity and use a spacetime diagram in your description and explanation.

[8 pt] ***b.*** Describe how two events may be simultaneous in one frame but not in another frame, again using a spacetime diagram to aid in your description and explanation.

[9 pt] ***c.*** Finally, describe and explain the Lorentz-Fitzgerald contraction using a spacetime diagram.

Problem 2.

For each of the wavefunctions $\psi_1(x,t) = 4\,\sin[2\pi(0.2\,x - 3\,t)]$ and $\psi_2(x,t) = 0.25\,\sin(7\,x - 3.5\,t)$ describing a system, determine the following:

[8 pt] ***a.*** frequency, wavelength and period, amplitude;

[8 pt] ***b.*** phase velocity, direction of motion.

[9 pt] ***c.*** Given that these wavefunctions satisfy the Schrödinger equation but are not stationary states, compute the potential to which the particle with the given wavefunction is subjected..

Problem 3.

A stationary wavefunction of a particle is known to be

$$\psi(x) = \begin{cases} A\,\sin\left(\frac{\pi x}{L}\right) e^{-x/L}, & \text{for} \quad x > 0, \\ 0 & \text{for} \quad x \leqslant 0, \end{cases}$$

[6 pt] ***a.*** Calculate the normalization constant, A.

[7 pt] ***b.*** Calculate the average value of the position of the particle.

[6 pt] ***c.*** Find the probability of finding the particle between $x = 0$ and $x = L$

[6 pt] ***d.*** Find the average value of the kinetic energy of the particle.

Problem 4.

Comsider a particle of mass m subject to a potential given as

$$V = +\infty \ \text{ for } x < 0, \qquad V = 0 \ \text{ for } 0 \le x < L, \qquad V = V_0 > 0 \ \text{ for } x > L.$$

[5 pt] ***a.*** Sketch the potential and specify all the boundary conditions.

[5 pt] ***b.*** Specify the allowed values of the total energy, and identify the range(s) of energies for which this system may have bound states, and for which range(s) it has scattering states.

[5 pt] ***c.*** For $0 < E < V_0$, solve the Schrödinger equation separately for $0 \le x \le L$ (region I), and $x > L$ (region II), and use the boundary conditions at $x = 0$ and $x = +\infty$ to determine two of the integration constants.

[5 pt] ***d.*** Impose the matching conditions at $x = L$, and obtain the transcendental equation which determines the allowed values of the total energy.

[5 pt] ***e.*** Compute the minimum value of V_0—if any—for which there will exist at least one bound state.

Problem 5.

Consider a linear harmonic oscillator with characteristic frequency ω.

[5 pt] ***a.*** Using the creation and annihilation operators, specify the Hamiltonian, its eigenvalues E_n, the allowed values of n, and specify how $\hat{a}$ and $\hat{a}^\dagger$ act on the stationary states, $|n\rangle$.

Assume that the harmonic oscillator also has a charge q and sonsider the effects of a constant electric field, $\vec{\mathcal{E}} = \mathcal{E}_0 \hat{x}$.

[6 pt] ***b.*** Compute the changes in the energies of all stationary states of the oscillator to the lowest non-zero order in perturbation theory.

[7 pt] ***c.*** Compute the changes in the energies of all stationary states of the oscillator exactly.

Consider now that the electric field is turned on at $t = 0$:

$$\vec{\mathcal{E}} = 0 \ \text{ for } t < 0, \qquad \text{but} \qquad \vec{\mathcal{E}} = \mathcal{E}_0 \sin(\omega' t)\hat{x} \text{ for } t \ge 0.$$

[7 pt] ***d.*** Compute the probability that after some finite time $t = T$, the system may be found in the state $|1\rangle$ if it was in $|0\rangle$ before $t = 0$.

Problem 6.

Consider the noninteracting nonrelativistic gas of a large number, N, of spin-$\frac{1}{2}$ electrons in a 3-dimensional slab of metal, of volume V.

[6 pt] ***a.*** Starting from the number of states accessible to a free particle with absolute value of its momentum being between p and $p+\mathrm{d}p$, and using Pauli's principle, compute the Fermi energy, E_F, in terms of N, V, m_e and $\hbar$ only.

[6 pt] ***b.*** Compute the total energy, E, in terms of N, V, m_e and $\hbar$ only.

[7 pt] ***c.*** Compute the pressure, P, in terms of N, V, m_e and $\hbar$, and combining with the previous results show that $PV = \frac{2}{3}E$.

[6 pt] ***d.*** Retrace the above derivation, but for ultra-relativistic electrons, for each of which energy and momentum are related by $\varepsilon = pc$, and show that now $PV = \frac{1}{3}E$, same as for photons.

Problem 7.

Consider N independent quantum harmonic oscillators with the Hamiltonian given as

$$H(n_i) = \sum_{i=1}^{N} \hbar\omega(n_i + \tfrac{1}{2})$$

[8 pt] ***a.*** Calculate the entropy, S, as a function of the total energy, E.

[8 pt] ***b.*** Calculate the energy, E, and the heat capacity, C, as functions of temperature, T, and the number of oscillators, N.

[9 pt] ***c.*** Find the probability, $P(n)$ that a particular oscillator is in its n^{th} quantum state.