The Howard University
Department of Physics and Astronomy

Master of Science Comprehensive and Doctor of Philosophy Qualifying Exam

Written exam: Classical Physics

August 23, 2011

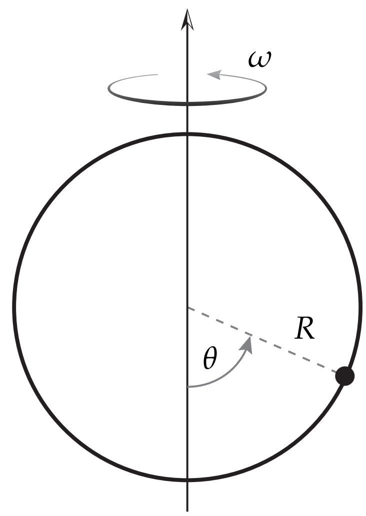
Attempt to solve ***four*** problems, at least one from each group. Circle the numbers below to indicate the problems you chose to solve:

1 2	3 4 5	6 7
Group A	Group B	Group C

1. Write in your chosen code-letter here: ☐
2. Write your code-letter and a page number (in sequential order) on the *top right-hand* corner of each submitted answer sheet.
3. Write only on one side of your answer sheets.
4. Start each problem on a new answer sheet.
5. Stack your answer sheets by problem and page number, and then staple them (at the top *left-hand corner*) with this cover sheet on the top.

Problem 1.

A bead of mass m is free to slide along a frictionless massless hoop of radius R, in a uniform vertical gravitational field. The hoop rotates with constant angular speed ω around a vertical diameter, as in the illustration to the right.



[9 pt] ***a***. Write the Lagrangian of the mass.

[8 pt] ***b***. Find the equations of motion for the angle θ, defined as shown in the illustration.

[8 pt] ***c***. What are the equilibrium positions? Are stable or unstable? Discuss the cases $\omega^2 < g/R$ and $\omega^2 > g/R$.

Problem 2.

Consider a particle of mass m constrained to move on the surface of a cylinder of radius R. The particle is subjected to a force directed toward the origin and proportional to the distance from the origin, $F = -kr$.

[8 pt] ***a***. Find the Hamiltonian of the particle.

[6 pt] ***b***. Discuss the occurrence of cyclic coordinates and constants of motion.

[6 pt] ***c***. Obtain the canonical equations.

[5 pt] ***d***. Find the equations of motion.

Problem 3.

N weakly-coupled particles obeying Maxwell-Boltzmann statistics may each exist in one of three non-degenerate energy levels of energies $-\epsilon, 0$ and ϵ. The system is in contact with a thermal reservoir at temperature T.

[5 pt] ***a***. What is the entropy of the system at $T = 0$ K?

[4 pt] ***b***. What is the maximum possible entropy of the system?

[4 pt] ***c***. What is the minimum possible energy of the system?

[4 pt] ***d***. What is the canonical partition function of the system?

[4 pt] ***e***. What is the most probable energy of the system?

[4 pt] ***f***. If $C(T)$ is the heat capacity of the system, what is the value of $\int_0^\infty \frac{C(T)}{T} dT$?

Problem 4.

Consider a collection of N two-level systems in thermal equilibrium at a temperature T. Each system has only two states: a ground state of energy 0 and an excited state of energy ϵ.

[7 pt] ***a***. Obtain the probability that a given system will be found in the excited state.

[6 pt] ***b***. Sketch the temperature dependence of this probability.

[7 pt] ***c***. Find the entropy of the entire collection of two-level systems.

[5 pt] ***d***. Sketch its temperature dependence.

Problem 5.

Consider an adsorbent surface having N sites, each of which can adsorb one gas molecule. Suppose the surface is in contact with an ideal gas with the chemical potential μ (determined by pressure p and temperature T). Assume that an adsorbed molecule has energy $-\epsilon_0$ compared to one in a free state.

[5 pt] ***a***. Find the number of possible ways of distributing N_1 identical molecules among N adsorbing sites.

[4 pt] ***b***. Write down the canonical partition function Z_{N_1}.

[4 pt] ***c***. Obtain the grand partition function Q.

[4 pt] ***d***. What is the probability $Pr(N_1)$ of N_1 molecules being adsorbed?

[4 pt] ***e***. Obtain the mean value N_1 of the number of adsorbed molecules.

[4 pt] ***f***. Determine the covering ratio $\theta = (N_1/N)$.

Problem 6.

Consider a long cylindrical wire of radius a carrying a steady uniform current I in the $\hat{e}_z$ direction. The conductivity of the material is σ. (Briefly indicate your choice of coordinates.)

[5 pt] ***a***. Find the electric field vector $\vec{E}$ inside the wire.

[5 pt] ***b***. Find the magnetic field $\vec{B}$ inside and outside the wire.

[5 pt] ***c***. Find the Poynting vector inside and (just barely) outside the wire; assume the electric field just outside the wire is the same as the field inside.

[5 pt] ***d***. Find the Poynting vector at the surface of the wire, and the energy flux through the surface, for a segment of length L.

[5 pt] ***e***. Compare the result of part (d) with the Joule heating (*i.e.* power dissipated) in the wire and comment on the physical significance.

Problem 7.

Consider a uniform plane wave in a vacuum whose electric field takes the following form:

$$\vec{E}(\vec{r},t) = E_0\,(3\hat{e}_x + \alpha\hat{e}_y)\cos\left(\frac{x-y+2z}{L} - \omega t\right) \tag{1}$$

[5 pt] ***a***. In terms of L: what is the wavelength λ? What is the angular frequency ω?

[5 pt] ***b***. Write down the vacuum Maxwell equations. Use them to determine α.

[5 pt] ***c***. Use the vacuum Maxwell equations to determine the magnetic field $\vec{B}$ of this plane wave.

[5 pt] ***d***. What are the electric and magnetic energy densities?

[5 pt] ***e***. What is the Poynting vector for this wave?

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August 25, 2011

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Problem 1.

Consider a dilute diatomic gas whose N molecules consist of non-identical pairs of atoms. The moment of inertia about an axis through the molecular center-of-mass, perpendicular to the line connecting the two atoms, is I.

[6 pt] ***a.*** Write an expression for the rotational energy ϵ_J of a single molecule in terms of the quantum number J.

[6 pt] ***b.*** Obtain the total canonical partition function Z for the system.

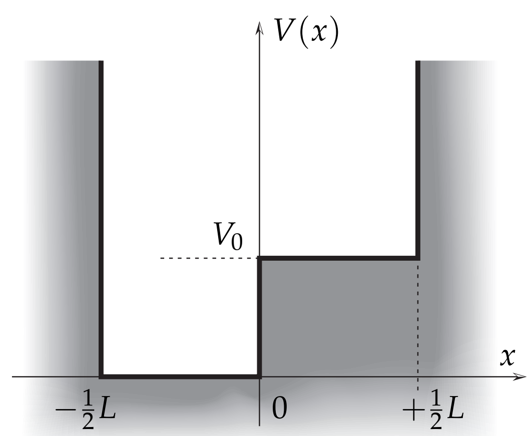
[7 pt] ***c.*** Determine the rotational energy U_R, as well as contributions to the heat capacity C_R and the entropy S_R per mole at temperature T.

[6 pt] ***d.*** Derive expressions for U_R, C_R and S_R for two limiting cases: (i) $k_B T \gg \hbar^2/I$ (ii) $k_B T \ll \hbar^2/I$.

Problem 2.

A particle of mass M moves in one dimension, where it is bound by an infinite potential well of width L that is divided into two equal parts, as illustrated to the right. On the left side the potential is zero, on the right side the potential is $V_0 > 0$. Everywhere else the potential is infinite. The initial wave function for the particle is given by the form

$$\psi(x) = \begin{cases} A_1 \sin(8\pi x/L) & \text{for } -\frac{L}{2} < x < 0 \\ A_2 \sin(4\pi x/L) & \text{for } 0 < x < \frac{L}{2}. \end{cases}$$



[7 pt] ***a.*** Under what condition(s) on V_0, M and L is this an eigenfunction of the energy for the 1D Schrödinger equation?

[6 pt] ***b.*** Determine the values of A_1 and A_2 that give a normalized energy eigenfunction of the given form.

[6 pt] ***c.*** What is its energy?

[6 pt] ***d.*** What is the probability that the particle is in the right half of the well?

Problem 3.

A rigid, diatomic molecule with moment of inertia I is known to be in a state

$$\psi(r,\theta,\phi,0) = \frac{R(r)}{\sqrt{26}}\left(3Y_1^1(\theta,\phi) + 4Y_7^3(\theta,\phi) + Y_7^1(\theta,\phi)\right)$$

where $R(r)$ is a fixed (and normalized) radial function and $Y_l^m(\theta,\phi)$ are the usual spherical harmonics.

[10 pt] ***a.*** Calculate the measured values of $L = \sqrt{L^2}$ and L_z in this state and the probabilities with which these values occur.

[8 pt] ***b.*** Calculate $\psi(r,\theta,\phi,t)$ at $t > 0$.

[7 pt] ***c.*** Calculate the expectation value of energy, $\langle E \rangle$, for this molecule.

Problem 4.

Consider a particle of mass m restricted to move in one dimension, bound by the potential $V(x) = -\frac{\lambda}{x}$ for $x > 0$ and $V(x) = +\infty$ for $x \leq 0$, with $\lambda > 0$ a real appropriate constant.

[4 pt] ***a.*** Write down the Schrödinger equation and specify/state all the boundary conditions.

[5 pt] ***b.*** Motivated by the Hydrogen atom, determine α so that $\psi_\infty(x) = e^{-(x/\alpha)}$ would approximately solve the Schrödinger equation when $x \gg \alpha$.

[10 pt] ***c.*** Writing $\psi(x) = f(x)e^{-(x/\alpha)}$, determine the differential equation for $f(x)$. Show that physically acceptable solutions must be polynomials with $f(0) = 0$, and then compute the bound state energy levels ($E < 0$) for which $\psi(x)$ satisfies all boundary conditions.

[6 pt] ***d.*** Compute $\langle\, \psi \,|\, x \,|\, \psi \,\rangle$ for $\psi_0(x) = N\,x\,e^{-(x/\alpha)}$, with the value of α you computed in part b), and N the proper normalization constant. Is $\psi_0(x)$ a stationary state? Is there a stationary state of energy lower than $\psi_0(x)$? Justify your answers.

Problem 5.

A 3-dimensional harmonic oscillator of frequency ω, is placed in a weak electric field $\vec{\mathcal{E}} = \mathcal{E}_0\,\hat{\mathrm{e}}_z$, and the harmonic oscillator itself has the electric charge q.

[7 pt] ***a.*** In the $\mathcal{E}_0 \to 0$ limit and using Cartesian coordinates, specify the energies of the stationary states of this oscillator, as well as the corresponding state-vectors using the creation/annihilation operators in the $x-, y-$ and $z-$direction.

[3 pt] ***b.*** Specify the degeneracies of at least the lowest-lying five energy levels.

[6 pt] ***c.*** Compute, to lowest non-vanishing perturbative order, the correction to all energy levels of this oscillator stemming from the $\vec{\mathcal{E}}$-field.

[6 pt] ***d.*** Compute the correction to all energy levels of this oscillator stemming from the $\vec{\mathcal{E}}$-field *exactly*, and compare with your perturbative result.

[3 pt] ***e.*** Does the $\vec{\mathcal{E}}$-perturbed oscillator still have any degeneracy? Explain.

Problem 6.

Consider a spin-1/2 quantum system in a mixed state represented by the state operator $\hat{\boldsymbol{\rho}}$.

[10 pt] ***a.*** Show that the most general form of $\hat{\boldsymbol{\rho}}$ is $\frac{1}{2}(a_0\mathbb{1} + \vec{a}\cdot\vec{\boldsymbol{\sigma}})$, where $\vec{\boldsymbol{\sigma}} = \sum_{i=1}^{3}\boldsymbol{\sigma}_i\,\hat{\mathrm{e}}_i$ and $\boldsymbol{\sigma}_i$ are the Pauli matrices. Show that $a_0 = 1$ and $\vec{a}$ is a real vector with $\vec{a}^{\,2} \leq 1$.

[5 pt] ***b.*** If the magnetic moment of this system is $\vec{\mu} = \frac{1}{2}\gamma\hbar\vec{\boldsymbol{\sigma}}$, specify the energy (Hamiltonian) that a constant magnetic field, $\vec{B}$, adds to this system.

[10 pt] ***c.*** Calculate the time-dependent state-operator $\hat{\boldsymbol{\rho}}(t)$ in the Shrödinger picture, i.e., describe the effect of the magnetic field on this system in terms of the time-evolution of $\vec{a} = \vec{a}(t)$.
(Hint: $\boldsymbol{\sigma}_j\boldsymbol{\sigma}_k = \delta_{jk}\mathbb{1} + i\varepsilon_{jk}{}^{\ell}\boldsymbol{\sigma}_\ell$ for $j,k,\ell = 1,2,3$ may be useful; $\varepsilon_{jk}{}^{\ell}$ is the Levi-Civita alternating symbol.)