Rotation and Torque (Equilibrium of Rigid Bodies)

Object: To study the use of a balanced meter stick, the concept of torque and the conditions that must be met for a body to be in rotational equilibrium.

Theory: When a rigid body with a fixed pivot point O, is acted upon by a force, there may be a rotational change in velocity of the rigid body. In the diagram below, there is a force $\vec{F}$ that is applied to the arm of the lever. The position relative to the pivot point O is defined by a vector $\vec{r}$. The two make an angle with each other φ ($\vec{F}$ is in the plane of the paper). $\vec{F}$ may be resolved into two components. The radial component F_r points along $\vec{r}$ and does not rotate. The tangential component $F_t$is perpendicular to $\vec{r}$ and has the magnitude $F_t = F * \sin\varphi$. This component does cause rotation. The quantity τ is defined as

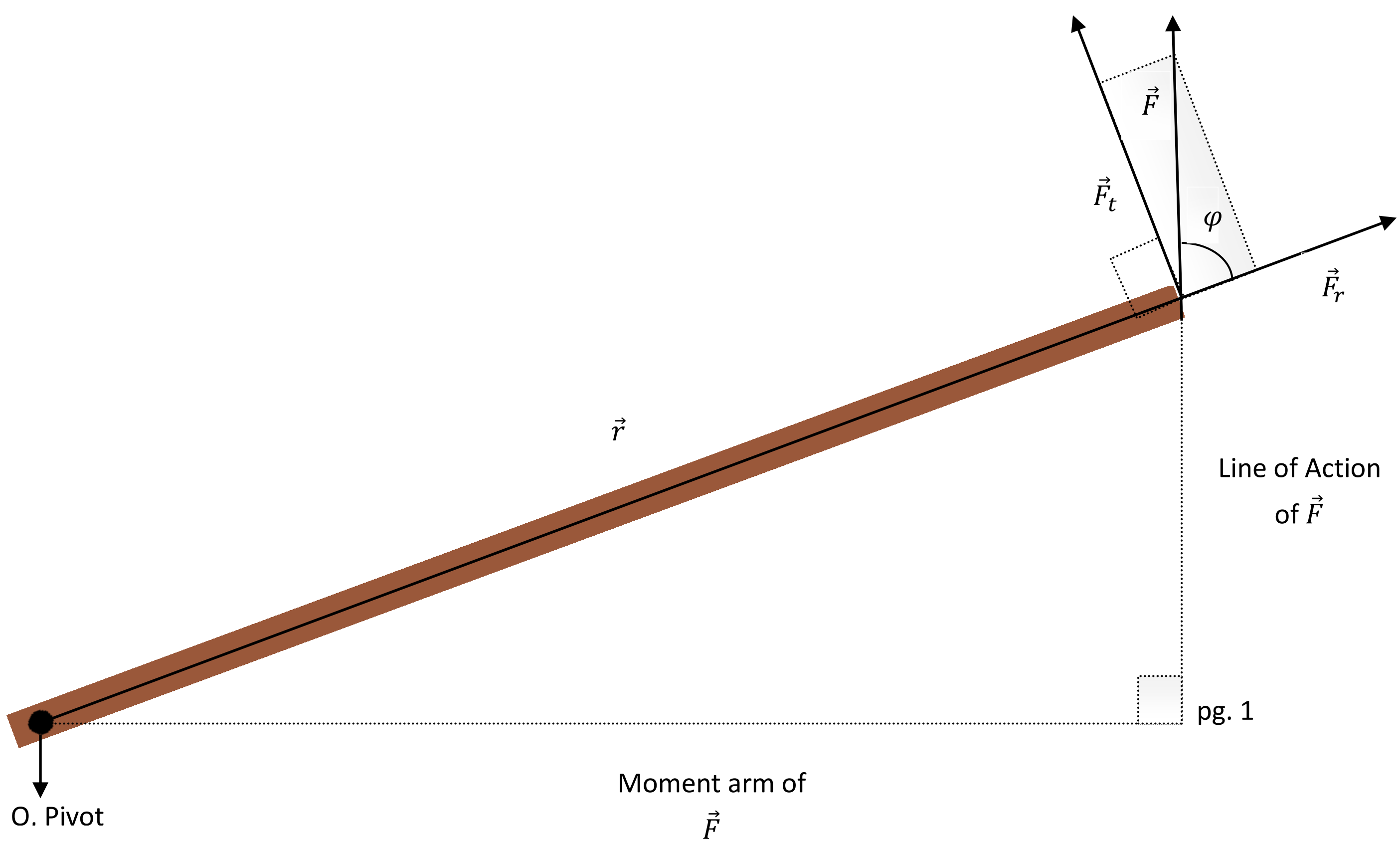
$$\tau = (r)(F * \sin\varphi)$$

There are two other ways of computing torque,

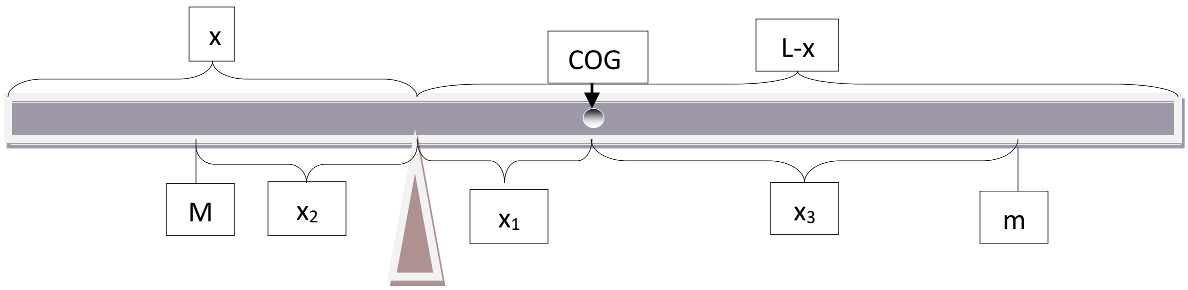
$$\tau = (r)(F * \sin\varphi) = r * F_t$$

$$\tau = (r * \sin\varphi)(F) = r_{\perp} * F$$

Where we have $r_{\perp}$ as a perpendicular distance between the rotation axis at the pivot and a line extended from $\vec{F}$. The extended line is the line of action of $\vec{F}$ and $r_{\perp}$ is the moment arm of $\vec{F}$.Torque means "to twist" and has the unit of $\mathrm{N} * \mathrm{m}$. Do not confuse the units of torque with the units of energy as Joule, J is defined as $N * m$. Torque is defined as positive if rotating counter-clockwise and negative if clockwise. Torque obeys the superposition principle, therefore when several torques act on a body, the net torque or resultant torque is the sum of the individual torques.



The gravitational force on an extended body is the vector sum of the gravitational forces acting on the individual elements of the body. The gravitational force $\vec{F}_g$ on a body effectively acts at a single point, called the center of gravity (COG) of the body. If $\vec{g}$ is the same for all elements of a body, then the body's center of gravity (COG) is coincident with body's center of mass (COM). We introduce λ as the linear density, where $\lambda = \frac{Length}{Mass}$.



Part II

$$\tau_L = \tau_R$$

$$r_1 F_1 = r_2 F_2$$

$$M * x_2 * g = M_{stick} * x_1 * g$$

$$M_{stick} = \frac{M * x_2}{x_1}$$

Second method:

$$\tau_L = \tau_R$$

$$r_1 F_1 = r_2 F_2$$

$$x * \left(\lambda * \frac{x}{2}\right) * g + M * x_2 * g = (L - x)\left(\lambda * \frac{(L - x)}{2}\right) * g$$

$$M_{stick} = \frac{2 * M * x_2}{L - 2 * x}$$

Part III

$$\tau_L = \tau_R$$

$$M x_2 g = M_{stick} x_1 g + m(x_3 + x_1) g$$

$$m = (M x_2 - M_{stick} x_1)/(x_3 + x_1)$$

Apparatus: Meter stick, stand, weight pans, sets of masses, and an unknown weight

Procedure:

Part I

1. Determine the center of gravity by balancing the meter stick on a sharp edge. Repeat several times. Use the value that is the average of the measured values.
2. Weigh the weight holders and label them. Use the average of several readings.

Center of Gravity of the Meter Stick

P_1	P_2	P_3	P_4	P_5	Average Position

Part II

1. Balance the stick on a sharp edge when a known mass of 50 g is hung from the stick. Do not use the center of gravity of the stick as the balance point or fulcrum and make sure that the fulcrum is always to the left of the COG, otherwise the equation does not hold. Determine the distance from the center of gravity of the stick to the new fulcrum. Determine the distance from the known mass to the new fulcrum. From the condition of equilibrium for torques, solve for the mass of the meter stick. Repeat at least three times with the 50 g mass hung from different places on the stick. Determine the average and compare it to the value determined by weighing the stick.

	Positions		**Distance to Fulcrum**		
	Fulcrum	**50 g**	**COG**(x_1)	**50 g** (x_2)	**Mass of Meter Stick**
1					
2					
3					
4					
5					

Average mass of meter stick: ________

Part III

1. Hang a 75 g mass from one hanger and an unknown mass from the second hanger. Balance the meter stick at a position other that the center of gravity and make sure that the fulcrum is always to the left of the COG, otherwise the equation does not hold. Determine the distances from the weight hangers to the fulcrum. Determine the unknown weight using the conditions of equilibrium.

	Positions			**Distance to Fulcrum**			
	Fulcrum	**75 g**	**Unknown**	**COG** (x_1)	**75 g** (x_2)	**Unknown**(x_1+x_3)	**Mass of Unknown Weight**
1							
2							
3							
4							
5							

Average mass of Unknown Weight: ________

Questions:

1. Why was the supporting force exerted on the meter stick by the sharp edge not considered in your calculations?
2. A meter stick is pivoted at its 50 cm mark but does not balance because of non-uniformities in its material that cause its center of gravity to be displaced from its geometrical center. However, when weights of 150 g and 300 g are placed at the 10 cm and 75 cm marks, respectively, balance is obtained. The weights are then interchanged and balance is again obtained by shifting the pivot point to the 43 cm mark. Find the mass of the meter stick and the location of its center of gravity.